We will now derive a third area function $A(\theta)$. Let $f(x) = (x)$. We have shown that for any $x \geq 0$, $f(\theta) \int_{\theta}^{\infty} f(x) dx = (x)$. We now derive a new function $A(y)$. We have shown that for any $y \geq 0$, $f(y) \int_{y}^{\infty} f(x) dx = (y)$. We need to answer the following question:

What is the area function $A(\theta)$?

1. Consider the area function $A(y)$ to calculate the area under the curve $f \int_{y}^{\infty} f(x) dx$. The area function $\int_{\theta}^{\infty} f(x) dx$ for $\theta = 0$.
2. Consider the function:

\[ f(x) = 1 - e^{-x} \]

3. Compute the derivative:

\[ f'(x) = e^{-x} \]

4. We are trying to derive one final function, \( g(x) \):

\[ g(x) = \int e^{-x} dx \]

5. We have now computed these different area functions. Fill in the blanks:

<table>
<thead>
<tr>
<th>Function</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>( (x) \int f(x) dx )</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>( (x) g(x) )</td>
</tr>
<tr>
<td>( h(x) )</td>
<td>( (x) h(x) )</td>
</tr>
</tbody>
</table>

6. The problem:

Explain the following integrals:

\[ \int e^{-x} dx \]

Explain why it is the case that:

\[ \int e^{-x} dx = (x) e^{-x} + C \]