Example 1 Use the formula for the area of a triangle to evaluate $\int_{-3}^{3} (x + 1) \, dx$.

Answer: Figure A1 $\bullet \int_{-3}^{3} (x + 1) \, dx = 6.$

Example 2 Calculate the right Riemann sum for $\int_{0}^{1} x^2 \, dx$ corresponding to the partition of $[0, 1]$ into five equal subintervals. Draw the curve $y = x^2$ with the rectangles whose areas give the Riemann sum.

Answer: Figure A2 $\bullet \text{[Right Riemann sum]} = 0.44$
Example 3  Use the fact that the curve $y = \sqrt{16 - x^2}$ is the upper half of the circle $x^2 + y^2 = 16$ of radius 4 with its center at the origin to find the exact value of $\int_{-4}^{0} \sqrt{16 - x^2} \, dx$.

Answer: $\int_{-4}^{0} \sqrt{16 - x^2} \, dx = 4\pi$

Example 4  Use five rectangles of equal width to find the approximate value of $\int_{0}^{5} H(x) \, dx$ for the function $y = H(x)$ of Figure 1.

Answer: One answer: $\int_{0}^{5} H(x) \, dx \approx (5 + 7 + 2.5 - 5 + 4)(10) = 135$