Homework 8 answers (even numbered problems).

11.10: 11.10.2. (a) Using Formula 6, a power series expansion of \( f \) at 1 must have \( f(1) + f'(1)(x-1) + \ldots \). Comparing to the given series, \( 1.6 - 0.8(x-1) + \ldots \), we must have \( f'(1) = -0.8 \). But from graph, \( f'(1) \) is a positive. Hence, the given series is not the Taylor series of \( f \) centered at 1.

(b) A power series expansion of \( f \) at 2 must have the form \( f(2) + f'(2)(x-2) + \frac{1}{2} f''(2)(x-2)^2 + \ldots \). Comparing to the given series, \( 2.8 + 0.5(x-2) + 1.5(x-2)^2 + 0.1(x-2)^3 + \ldots \), we must have \( \frac{1}{2} f''(2) = 1.5 \); that is \( f''(2) \) is a positive. But from the graph, \( f \) is concave downward near \( x = 2 \), so \( f''(2) \) must be negative. Hence, the given series is not the Taylor series of \( f \) centered at 2.

11.10.62a. \( f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \)

\[ f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{e^{-1/x^2}}{x} = \lim_{x \to 0} \frac{1/x}{e^{1/x^2}} = \lim_{x \to 0} \frac{x}{2e^{1/x^2}} = 0 \] (using l’Hospital’s Rule and simplifying in the penultimate step). Similarly, we can use the definition of the derivative and l’Hopital’s Rule to show that \( f''(0) = 0, f^{(3)}(0) = 0, \ldots, f^{(n)}(0) = 0 \), so that the Maclaurin series for \( f \) consists entirely of zero terms. But since \( f(x) \neq 0 \) except for \( x = 0 \), we see that \( f \) cannot equal its Maclaurin series except at \( x = 0 \).